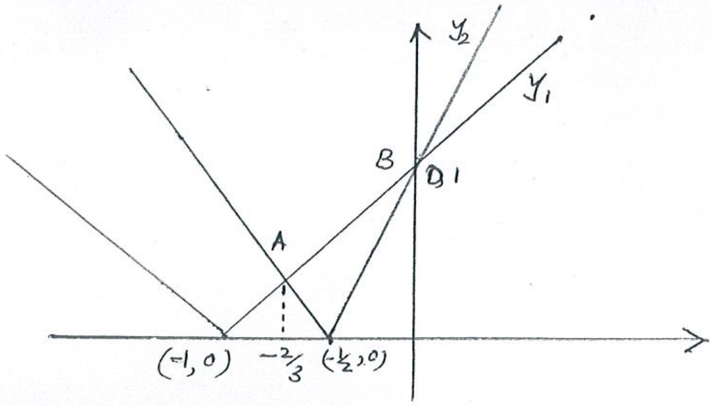
(1) $y_1 = \begin{cases} x+1 & x+1 \geqslant 0 \\ -x-1 & x < -1 \end{cases}$

$y_2 = \begin{cases} 2x+1 & x \geqslant -\frac{1}{2} \\ -2x-1 & x < \frac{1}{2} \end{cases}$



A $\quad x+1 = -2x-1$

$x = -\frac{2}{3}$

B $\quad x+1 = 2x+1$

$x = 0$

$y_1 < y_2$

$-\infty < x < -\frac{2}{3} \cup 0 < x < \infty$

(2) $E-2 \quad C-3 \quad N-1 \quad R-1 \quad T-3 \quad I-1 \quad Y-1$

(i). $\dfrac{12!}{3!\,3!\,2!}$

(ii) $\cancel{NT}$ $\quad T, \quad {}^3C_1 = 3$

$$\frac{11!}{2!\,2!\,3!} \times {}^3C_1 = \frac{11! \times 3}{2!\,2!\,3!} = \frac{11!}{(2!)^3}$$

(3) $(1-x+x^2)\left({}^{2012}C_1 + {}^{2012}C_1 x + \cdots + {}^{2012}C_{2011} x^{2011} + {}^{2012}C_{2012} x^{2012}\right)$

$$\left({}^{2012}C_{2011} - {}^{2012}C_{2012}\right) x^{2013} = 2011\, x^{2013}$$

2011

(4) (i) $\lim\limits_{x \to 0} \dfrac{\sqrt{1+x+x^2} - 1}{x} = \lim\limits_{x \to 0} \dfrac{x+x^2}{x(\sqrt{1+x+x^2} + 1)}$

$= \dfrac{1}{2}$

(ii) $\lim\limits_{x \to \infty} \dfrac{1+x-3x^2}{x^2-4x+8} = \lim\limits_{x \to \infty} \dfrac{\frac{1}{x^2} + \frac{1}{x} - 3}{1 - \frac{4}{x} + \frac{8}{x^2}}$

$= -3$

(5) $\ln y + \frac{1}{x+1} \ln 3 = \ln 4$

$$\frac{1}{y}\frac{dy}{dx} - \frac{1}{(x+1)^2}\ln 3 = 0$$

$$\frac{dy}{dx} = \frac{\ln 3}{(x+1)^2} \times \frac{4}{3^{\frac{1}{x+1}}}$$

---

(6) $\int_{-2}^{2} |x+1|\, dx = \int_{-2}^{-1} (-x-1)\, dx + \int_{-1}^{2} (x+1)\, dx$

$$= \underline{\underline{5}}$$

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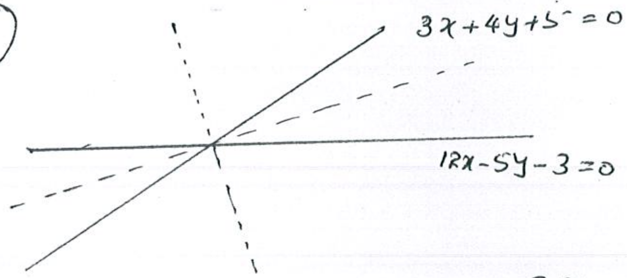
(7)



$$\frac{3x+4y+5}{5} = \pm \frac{12x-5y-3}{13}$$

(i) $3x - 11y - \frac{80}{7} = 0$ (ii) $11x + 3y + \frac{50}{9} = 0$

(0,0) සිට $\perp$ $\frac{3 \cdot 0 - 11 \times 0 - \frac{80}{7}}{\sqrt{130}} = -\frac{80}{7\sqrt{130}}$

(0,0) සිට $\perp$ $12x - 5y - 3 = 0$, $-\frac{3}{13}$

$\therefore$ Equation. $3x - 11y - \frac{80}{7} = 0$ //

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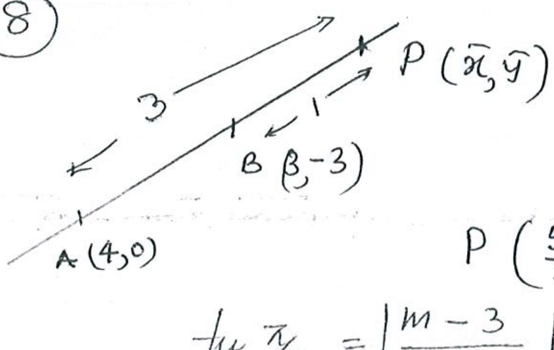
(8)



$\bar{x} = \frac{3 \times 3 - 1 \times 4}{3-1} = \frac{5}{2}$

$\bar{y} = \frac{3(-3) - 1 \times 0}{3-1} = -\frac{9}{2}$

$P\left(\frac{5}{2}, -\frac{9}{2}\right)$

AB $-$ $\frac{0-(-3)}{4-3} = 3$

$$\tan \frac{\pi}{4} = \left| \frac{m-3}{1+3m} \right|$$

$m = -2$ or $m = \frac{1}{2}$

$\frac{y + \frac{9}{2}}{x - \frac{5}{2}} = -2$ or $\frac{y + \frac{9}{2}}{x - \frac{5}{2}} = \frac{1}{2}$

$\underline{\underline{2y + 4x - 1 = 0}}$ $\underline{\underline{4y - 2x + 23 = 0}}$

(9) $x^2+y^2+2gx+2fy+c=0$ center $(-g,-f)$

radius $=\sqrt{g^2+f^2-c}$

$y=0$ $\sqrt{g^2+f^2-c} = |f|$

$g^2 = c$

$x=0$ $\sqrt{g^2+f^2-c} = |g|$

$f^2 = c$ $g^2 = f^2 \Rightarrow g = \pm f$

$$\left|\frac{-4g+3f-6}{5}\right| = \sqrt{g^2+f^2-c}$$

$$\left|\frac{-5g+3f-6}{5}\right| = f$$

$f=g$ $\begin{cases} g=-1 & f=-1 & c=1 \\ g=\frac{3}{2} & f=\frac{3}{2} & c=\frac{9}{4} \end{cases}$

~~$f=-g$~~

$f=-g$ $\begin{cases} g=-3 & f=3 & c=9 \\ g=-\frac{1}{2} & f=\frac{1}{2} & c=\frac{1}{4} \end{cases}$

$x^2+y^2-2x-2y+1=0$

$x^2+y^2+3x+3y+\frac{9}{4}=0$

$x^2+y^2-6x+6y+9=0$

$x^2+y^2-x+y+\frac{1}{4}=0$

---

(10) $L.H.S. = \tan\frac{B}{2}\left(\frac{\sin\frac{A}{2}}{\cos A/2}+\frac{\sin\frac{C}{2}}{\cos C/2}\right)+\frac{\sin A/2 \sin C/2}{\cos A/2 \cos C/2}$

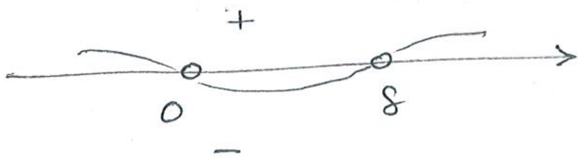
$$= \frac{\sin B/2}{\cos B/2}\;\frac{\cos B/2}{\cos A/2 \cos C/2} + \frac{\sin A/2 \sin C/2}{\cos A/2 \cos C/2}$$

$$= \frac{\cos\left(\frac{A+C}{2}\right) + \sin A/2 \sin C/2}{\cos A/2 \cos C/2}$$

$$= \frac{\cos A/2 \cos C/2}{\cos A/2 \cos C/2} = 1$$

11 (a) (i) $\Delta_x = (k-2)^2 - 4(k+1) > 0$

$= k(k-8) > 0$



$\underline{\underline{k < 0 \text{ . and } \quad k > 8}}$

(ii) $|\alpha+\beta| < 5$

$|k-2| < 5$

$-3 < k < 7$

(i) and ii $\quad \underline{\underline{-3 < k < 0}}$

(b) $$\frac{x^3+4x^2-10x+6}{(x-3)(x^2+4)} = A + \frac{B}{x-3} + \frac{Cx+D}{x^2+4}$$

$$= 1 + \frac{3}{x-3} + \frac{4x-2}{x^2+4}$$

---

12 $\quad U_n + U_{n-2} = 2U_{n-1} \qquad U_{n+1} - U_1 = nd$

$n=2 \quad$ L.HS $\frac{1}{U_1U_2} \qquad$ R.HS $= \frac{2-1}{U_1U_2} = \frac{1}{U_1U_2}$

$n=2$ is troue

$n=p \quad \sum_{r=2}^{p} \frac{1}{U_{r-1}U_r} = \frac{p-1}{U_1U_p}.$

$$\sum_{r=2}^{p} \frac{1}{U_{r-1}U_r} + \frac{1}{U_pU_{p+1}} = \frac{p-1}{U_1U_p} + \frac{1}{U_pU_{p+1}}$$

$$= \frac{1}{U_p}\left[\frac{p-1}{U_1} + \frac{1}{U_{p+1}}\right]$$

$$= \frac{1}{U_p}\left[\frac{pU_{p+1} - U_{p+1} + U_1}{U_1U_{p+1}}\right]$$

$$\sum_{r=2}^{p+1} \frac{1}{U_{r-1}U_r} = \frac{1}{U_p}\frac{pU_p}{U_1U_{p+1}}$$

$$= \frac{(p+1)-1}{U_1U_{p+1}}.$$

$n = p+1$ is troue

12 (b) $r(r+1) = Ar^2 + Br + C - 3\left[A(r-1)^2 + B(r-1) + C\right]$

$$A = -\frac{1}{2} \qquad B = -2 \qquad C = -\frac{9}{4}$$

$$\frac{V_r}{3^r} = \frac{f(r)}{3^r} - \frac{f(r-1)}{3^{r-1}}$$

$$\sum_{r=1}^{n} \frac{V_r}{3^r} = \frac{f(n)}{3^n} - \frac{f(0)}{1}$$

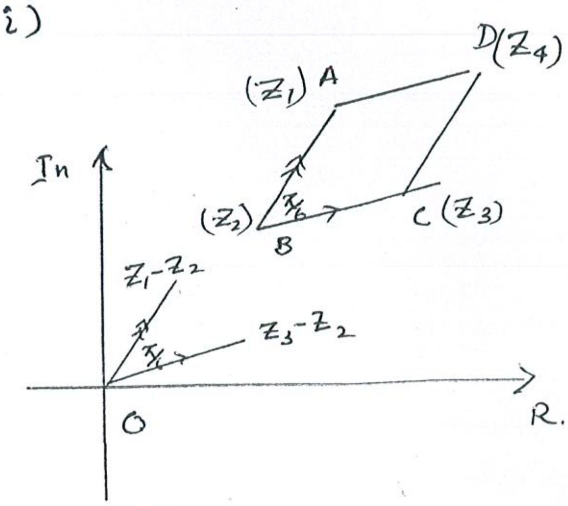
$$= \left(-\frac{n^2}{2} - 2n - \frac{9}{4}\right)\frac{1}{3^n} + \frac{9}{4}$$

$$\lim_{n\to\infty} \sum_{r=1}^{n} \frac{V_r}{3^r} = \infty$$

---

13 (a) $K = 14$ $\qquad A^{-1} = \frac{1}{6}\begin{pmatrix} 1 & 1 \\ 4 & -2 \end{pmatrix} = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} \\ \frac{2}{3} & -\frac{1}{3} \end{pmatrix}$

(b) (i) $$Z = \frac{(5+5i)(3+i)}{(-1+2i)(3-i)}$$
$$= \frac{13}{5} - \frac{9}{5}i$$



(ii)

$$Z_1 - Z_2 = (Z_3 - Z_2)\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$$

$$Z_1 - Z_2 = Z_3\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) - Z_2\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$$

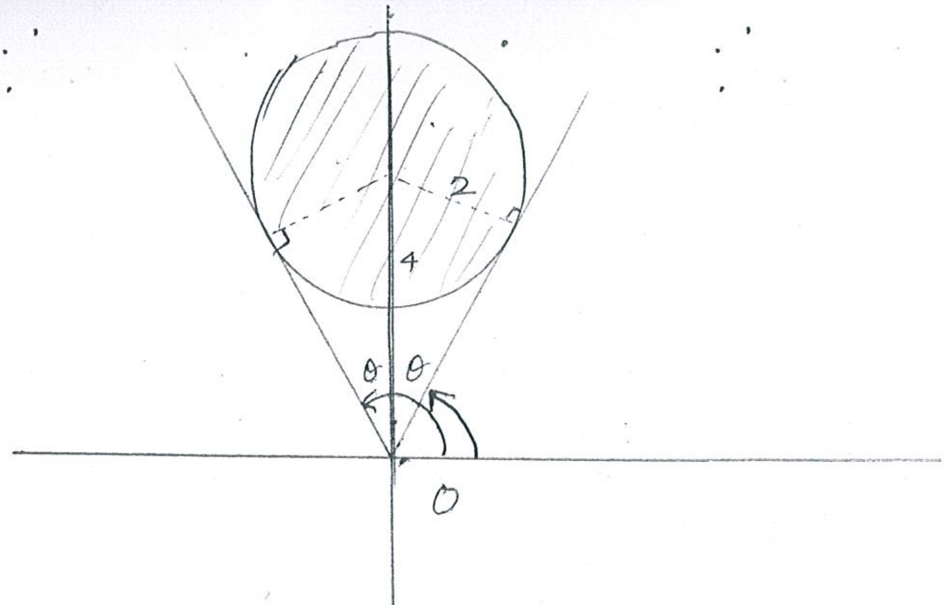
$$Z_2\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) = Z_3\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) - Z_1$$

$$Z_2 = \frac{1}{2}Z_1\left(1 + \sqrt{3}i\right) + \frac{1}{2}Z_3\left(1 - \sqrt{3}i\right)$$

$$Z_4 + Z_2 = Z_1 + Z_3$$

$$Z_4 = Z_1 + Z_3 - Z_2$$

$$= \frac{1}{2}Z_1\left(1 - \sqrt{3}i\right) + \frac{1}{2}Z_3\left(1 + \sqrt{3}i\right)$$



$$(\text{maximum } \arg z - \text{minimum } \arg z) = 2\theta$$

$$\sin\theta = \frac{2}{4} = \frac{1}{2} \qquad \theta = \pi/6$$

$$2\theta = \pi/3$$

$$|z|_{maxi} = 6$$

$$|z|_{mini} = 2$$

$$|z|_{maxi} - |z|_{mini} = 4$$

---

(14) (a) (i) $$a(x-y) = \sqrt{1-x^2} + \sqrt{1-y^2}$$

$$a(\sin\theta - \sin\alpha) = \cos\theta + \cos\alpha$$

$$2a\cos\frac{\theta+\alpha}{2}\sin\frac{\theta-\alpha}{2} = 2\cos\frac{\theta+\alpha}{2}\cos\frac{\theta-\alpha}{2}$$

$$\tan\frac{\theta-\alpha}{2} = \frac{1}{a}$$

$$\frac{\theta-\alpha}{2} = \tan^{-1}\left(\frac{1}{a}\right)$$

$$\theta - \alpha = 2\tan^{-1}\left(\frac{1}{a}\right)$$

$$\frac{d}{dx}(\sin^{-1}x - \sin^{-1}y) = \frac{d}{dx}\left(2\tan^{-1}(1/a)\right)$$

$$\frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$$

(14) (ii). $y = \dfrac{x\sin^{-1}x}{\sqrt{1-x^2}}$

$$\sqrt{1-x^2}\; y = x\sin^{-1}x$$

$$\sqrt{1-x^2}\frac{dy}{dx} - xy = x + \sqrt{1-x^2}\,\sin^{-1}x$$

$$(1-x^2)\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} = 2$$

$$(1-x^2)\frac{d^3y}{dx^3} - 5x\frac{dy}{dx} - 3\frac{dy}{dx} = 0$$

$x=0,\ y=0$ $\qquad$ $x=0,\ \dfrac{dy}{dx}=0$

$x=0,\ \dfrac{d^2y}{dx}=0$ $\qquad$ $x=0,\ \dfrac{d^3y}{dx^3}=0$

(b) $AB + BC = 4\cos\theta + 4\theta$

$$T(\theta) = \frac{2\cos\theta}{\sqrt{3}} + \theta$$

$$\frac{d}{d\theta}(T(\theta)) = 1 - \frac{2}{\sqrt{3}}\sin\theta$$

$\dfrac{d}{d\theta}(T(\theta)) = 0$ $\qquad$ $\sin\theta = \dfrac{\sqrt{3}}{2}$

$\theta = \pi/3$

$\theta < \pi/3$ $\quad \dfrac{d(T(\theta))}{d\theta} \geq 0$

$\theta > \pi/3$ $\quad \dfrac{d}{d\theta}(T(\theta)) < 0$

$T(\theta)$ Maximum, $\theta = \pi/3$

(15) (i) $\int_0^{\pi/2} \frac{\cos x}{3+\cos^2 x}\, dx$ $\sin x = t.$ $x=0 \quad t=0$

$\cos x\, dx = dt.$ $x=\pi/2 \quad t=1$

$= \int_0^1 \frac{dt}{4-t^2}.$

$= \frac{1}{4}\int_0^1 \frac{dt}{2+t} + \frac{1}{4}\int_0^1 \frac{dt}{2-t}$

$= \frac{1}{4}\Big[\ln|2+t| - \ln|2-t|\Big]_0^1$

$= \frac{1}{4}\ln 3$

(ii) $I_n = \int (a+x^2)^{-n}\, dx$

$= \frac{x}{(a+x^2)^n} + 2n\int \frac{x^2}{(a+x^2)^{n+1}}\, dx.$

$= \frac{x}{(a+x^2)^n} + 2n\int \frac{dx}{(a+x^2)^n} - 2an\int \frac{dx}{(a+x^2)^{n+1}}$

$I_n = \frac{x}{(a+x^2)^n} + 2n\, I_n - 2an\, I_{n+1}$

$2an\, I_{n+1} = (2n-1)\, I_n + \frac{x}{(a+x^2)^n}$

$I_{n+1} = \frac{x}{2an\,(a+x^2)^n} + \frac{2n-1}{2an} I_n$

$I_3 = \int \frac{1\, dx}{(4+x^2)^3}$

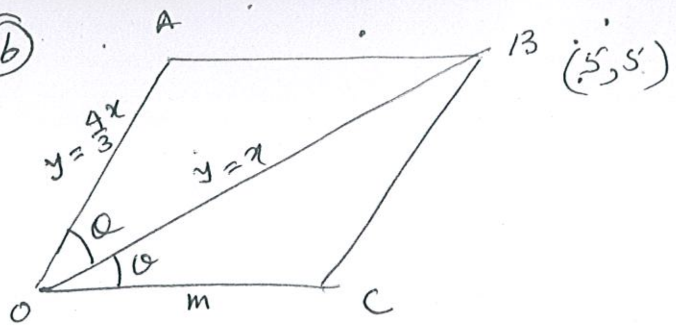
$a=4$ $I_3 = \frac{x}{2\times 4\times 2\,(4+x^2)^2} + \frac{3}{16} I_2$

$I_2 = \frac{I_1}{8} + \frac{x}{8(4+x^2)}$

$I_1 = \frac{1}{2}\tan^{-1}\frac{x}{2} + c$

$\therefore I_3 = \frac{x}{16(4+x^2)^2} + \frac{3}{16}\left[\frac{x}{8(4+x^2)} + \frac{1}{8}\left(\frac{1}{2}\tan^{-1}\frac{x}{2} + c\right)\right]$

(16)



$$\left(\frac{\frac{4}{3}-1}{1+\frac{4}{3}}\right) = \frac{1-m}{1+m}$$

$m = \frac{3}{4}$ $\qquad$ OC $\cancel{=}$ $y = \frac{3}{4}x$

$\underline{\underline{4y-3x=0}}$ — OC

AB — $\frac{y-5}{x-5} = \frac{3}{4}$ / $4y-3x+d=0$

$(5,5)$ $\quad d=-5$

$\underline{\underline{4y-3x-5=0}}$

AC — $x+y+k=0$

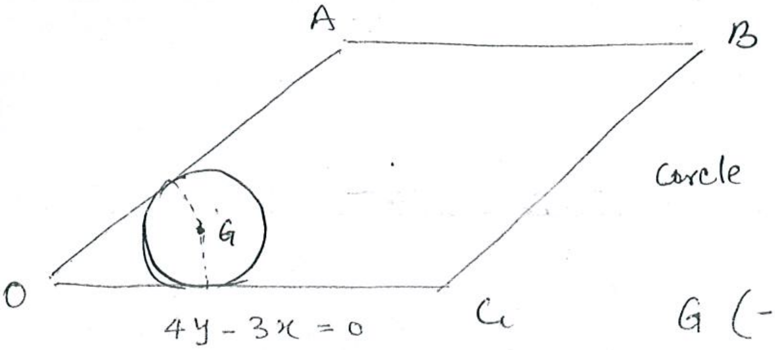
$(5/2, 5/2)$ $\quad k=-5$

$\underline{\underline{x+y-5=0}}$

BC $\quad 3y-4x+\lambda=0$

$(5,5)$ $\quad \lambda = 5$

$\underline{\underline{3y-4x+5=0}}$



Circle $x^2+y^2+2gx+2fy+c=0$

$f=g.$

$G(-g,-g)$

$r=1$

$g^2+g^2-c=1$

$c=49$

$$\left|\frac{-4g+3g}{\sqrt{16+9}}\right| = 1$$

$$\left|\frac{g}{5}\right| = 1$$

$g=5 \qquad g=-5$

$x^2+y^2+10x+10y+49=0$

$x^2+y^2-10x-10y+49=0$

Circle $x^2+y^2+10x+10y+49=0$

Tangent chord. $5x+5y+49=0$ or

$5x+5y-49=0$

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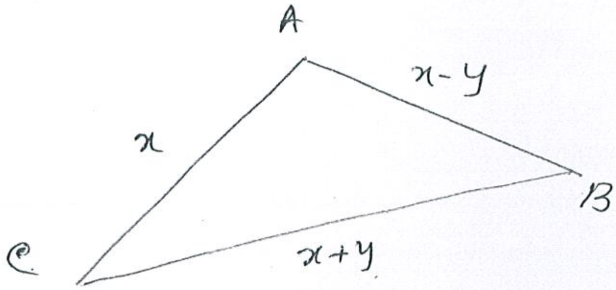
(17) $\cos A = \dfrac{b^2+c^2-a^2}{2bc}$



$$\cos A = \frac{(x-y)^2+x^2-(x+y)^2}{2x(x-y)}$$

$$= \frac{x^2-4xy}{2x(x-y)} = \frac{x-4y}{2(x-y)}$$

(II) $(\cos\alpha+\cos\beta)^2+(\sin\alpha+\sin\beta)^2 = 4\cos^2\frac{\alpha-\beta}{2}$

L.H.S. $= (\cos\alpha+\cos\beta)^2+(\sin\alpha+\sin\beta)^2$

$= 4\cos^2\frac{\alpha+\beta}{2}\cos^2\frac{\alpha-\beta}{2} + 4\sin^2\frac{\alpha+\beta}{2}\cos^2\left(\frac{\alpha-\beta}{2}\right)$

$= 4\cos^2\left(\frac{\alpha-\beta}{2}\right)$ = R.H.S.

$\therefore (\cos\alpha+\cos\beta)^2+(\sin\alpha+\sin\beta)^2 = 4\cos^2\left(\frac{\alpha-\beta}{2}\right)$

$(\cos x+\cos 3x)^2+(\sin x+\sin 3x)^2 = 4\cos^2\left(\frac{x-3x}{2}\right)$

$4\cos^2 x = 1$

$\cos x = \pm\frac{1}{2}$

$\cos x = 1/2$ $\quad\quad$ $\cos x = -1/2$

$x = \pi/3$ $\quad\quad$ $x = 2\pi/3$

$x = 2n\pi \pm \pi/3$ $\quad\quad$ $x = 2k\pi \pm 2\pi/3$

$n, k \in \mathbb{R}$